SECONDARY GRAVITATIONAL ANISOTROPIES IN OPEN UNIVERSES

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Abstract

The applicability of the potential approximation in the case of open universes is tested. Great Attractor-like structures are considered in the test. Previous estimates of the Cosmic Microwave background anisotropies produced by these structures are analyzed and interpreted. The anisotropies corresponding to inhomogeneous ellipsoidal models are also computed. It is proved that, whatever the spatial symmetry may be, Great Attractor-like objects with extended cores (radius $\sim 10h^{-1}$), located at redshift $z = 5.9$ in an open universe with density parameter $\Omega_0 = 0.2$, produce secondary gravitational anisotropies of the order of 10^{-5} on angular scales of a few degrees. The amplitudes and angular scales of the estimated anisotropy decrease as the Great Attractor size decreases. For comparable normalizations and compensations, the anisotropy produced by spherical realizations is found to be smaller than that of ellipsoidal models. This anisotropy appears to be an integrated effect along the

photon geodesics. Its angular scale is much greater than that subtended by the Great Attractor itself. This is understood easily taking into account that the integrated effect is produced by the variations of the gravitational potential, which seem to be important in large regions subtending angular scales of various degrees. As a result of the large size of these regions, the spatial curvature of the universe becomes important and, consequently, significant errors (~ 30 per cent) arise in estimations based on the potential approximation. As it is emphasized in this paper, two facts should be taken into account carefully in some numerical estimates of secondary gravitational anisotropies in open universes: (1) the importance of scales much greater than those subtended by the cosmological structures themselves, and (2) the compatibility of the potential approximation with the largest scales.

Subject headings: cosmic microwave background—cosmology: theory—large-scale structure of the universe

1   INTRODUCTION

Recently, the Tolman-Bondi solution of the Einstein equations (Tolman 1934; Bondi 1947) was used in order to estimate the Cosmic Microwave Background (CMB) anisotropies produced by Great Attractor-like (GAL) structures (Panek 1992; Sáez, Arnau & Fullana 1993; Arnau, Fullana & Sáez 1994; Fullana, Sáez & Arnau 1994; Sáez, Arnau & Fullana 1995). These structures were placed at a wide range of redshifts. The most interesting results appeared in the case of an open universe with density parameter $\Omega_0 \leq 0.4$. In this case, it was proved that GAL objects placed at redshifts between 2 and 30 produce CMB anisotropies having an amplitude of the order of 10^{-5} and an angular scale of a few degrees. For $\Omega_0 = 0.2$, the maximum amplitude corresponds to a redshift $z \sim 5.9$. What is the origin of these secondary anisotropies?.

Calculations based on the Tolman-Bondi solution seem not to be appropriate in order to answer the above question. These calculations are based on an numerical integration along the null geodesics of the Tolman-Bondi spacetime. Such a general and abstract method lead to accurate results, but it difficults the splitting of different posible effects contributing to the total anisotropy and, consequently, the origin of the predicted effect does not become clear.

In the case $\Omega_0 = 0.2$, it was verified that the density contrast of normalized GAL structures is close to unity at $z = 5.9$; namely, these structures were evolving in the mildly nonlinear regime when they influenced the CMB photons. This fact suggests

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the absence of strong nonlinear effects (see Rees & Sciama 1968 where some sources of nonlinear effects are described qualitatively). Furthermore, it was also verified that the anisotropy produced by a GAL object at $z = 5.9$ decreases strongly as the density parameter increases (Arnau, Fullana & Sáez 1994). After these considerations, it seems that we are concerned with anisotropies produced by the time variation of the gravitational potential (which depends on Ω_0 strongly). Nevertheless, such an interpretation is only a qualitative one. How can we obtain a quantitative verification of this interpretation?. What is the best formalism to do it?.

Let us try to answer these questions after some necessary words about notation. Hereafter, a is the scale factor, t is the cosmological time, and Ω is the density parameter. Whatever F may be, F_0, F_I, and F_B stand for the present, initial and background values of F, respectively. The universe is considered to be open and its present density parameter is fixed to be $\Omega_0 = 0.2$. The reduced Hubble constant is $h = H_0/100$, where H_0 is the Hubble constant in units of $Km\ s^{-1}Mpc^{-1}$. Latin indices run from 1 to 3. Coordinates x^i are pseudo-cartesian. In formulae, units are chosen in such a way that $8\pi G = c = 1$, where G and c are the gravitational constant and the speed of the light, respectively. Vector $\vec{v}$ stands for the peculiar velocity at an arbitrary point P, and $\vec{v}_r$ is the component of the peculiar velocity in the direction of the line joining the point P and the centre of the GAL object. The energy density and the energy density contrast are denoted ρ and δ, respectively.

In the case of objects located far from the observer and his last scattering surface,

the Sachs-Wolfe (1967) effect and the Doppler anisotropies produced by peculiar motions are negligible. On account of this fact, the anisotropy produced by these objects appears to be an integrated effect due to the variations of the gravitational potential (Martínez-González, Sanz & Silk 1990; Sanz et al. 1996). This anisotropy is given by the formula:

$$\frac{\Delta T}{T} \sim -2 \int_e^o \nabla \phi(x^i, t) dx^i \sim 2 \int_e^o \frac{\partial \phi(x^i, t)}{\partial t} dt \ , \tag{1}$$

where e and o stand for emitter and observer, respectively, ∇ is the gradient operator and ϕ is the potential involved in the line element

$$ds^2 = -(1 + 2\phi)dt^2 + (1 - 2\phi)a^2(1 + \frac{Kr^2}{4})^{-2}\delta_{ij}dx^i dx^j \ . \tag{2}$$

where K takes on the value -1 (1, 0) in the open (closed, flat) case and $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$. This potential satisfies the equation

$$\nabla^2 \phi = \frac{3}{2}H^2 a^2 \Omega \delta \ , \tag{3}$$

and, consequently, it can be interpreted as the Newtonian gravitational potential.

The integrals involved in Eq. (1) are to be carried out along a null geodesic from the emitter (e) to the observer (o). The equations of these geodesics can be derived in the background (at zero order). Although this approach was initially proposed in the framework of the flat case ($K = 0$, $\Omega_0 = 1$), Sanz et al. (1996) suppose it also valid for open universes in the case of systems having sizes much smaller than the curvature scale. This validity will be tested at the same time that we try to answer

the following questions: can we apply the potential approximation in the case of GAL objects evolving in open universes?. Do we expect some errors in the results?. A few considerations about sizes are necessary in order to consider these questions.

Calculations based on the exact Tolman-Bondi solution prove that, in the case $z = 5.9$, $\Omega_0 = 0.2$, the secondary anisotropy produced by a GAL object has an unexpected angular scale of a few degrees. This large angular scale is greater than the angular scale subtended by the GAL itself. This fact is not surprising, at least, if the resulting anisotropy is produced by the gradients of the gravitational potential. In such a case, the region affected by a significant gravitational potential is larger than that having a relevant density contrast (see Fullana, Sáez & Arnau 1994). At $z = 5.9$, scales subtending an angle between 6 and 10 degrees are spatial scales between 210 Mpc and 350 Mpc, while the curvature scale at the same redshift is $\sim 970\ Mpc$. This means that the regions in which the CMB photons are influenced by the GAL structure is between 20 and 30 per cent of the horizon scale and, consequently, the curvature could be relevant and the use of Eqs (1)-(3) could leads to significant errors. This suspicion will be confirmed by explicit calculations (see below). This fact is crucial in order to do further applications –including GAL objects– of general methods for the estimation of secondary gravitational anisotropies in open universes (see Section 4). Among these methods, the numerical approach used by Tului, Laguna & Aninos (1996) and the estimates based on spectra due to Sanz et al. (1996) deserve special attention.

Calculations based on an exact solution of the Einstein equations –as the Tolman-Bondi one– do not involve any gauge or approximating condition; hence, results are theoretically confident and, consequently, if the results obtained from the Tolman-Bondi solution and from the equations (1)-(3) become comparable, we would have given strong support to the following ideas: (1) the secondary significant effect calculated with Tolman-Bondi solution is a consequence of the time variations of the gravitational potential, (2) the approach based on Eqs. (1)-(3) applies in the open case up to a certain level of accuracy, and (3) calculations based on the Tolman-Bondi solution are both theoretically confident and well performed.

The main limitation of the estimations based on the Tolman-Bondi solution is expected to be a consequence of spherical symmetry. The time evolution of a given structure and, consequently, the time variations of its gravitational potential are conditioned by the symmetry. Hence, according to Eq (1), the anisotropy could be also affected by spherical symmetry.

The normalization of the Great Attractor is also strongly affected by this symmetry; in fact, what we know about this structure is the peculiar velocity that it produces on the sourrounding galaxies; in particular, following (Lynden-Bell et al. 1988), the Great Attractor produces a peculiar velocity $\mid \vec{v}_{r0} \mid = 570 \pm 60 Km \ s^{-1}$ at the radial distance $R_0 \sim 43h^{-1} \ Mpc$ corresponding to the distance from Local Group to the Great Attractor centre. The peculiar velocity field depends on the distance to the centre, R_0, as it corresponds to the velocity field created by an overdensity with

a density contrast proportional to $1/R_0^2$. These data and the size of the core radius were used in previous papers for normalizing GAL objects. It is evident that GAL structures as an elongated ellipsoid or a pancake would produce different peculiar velocities at different points located at $43h^{-1}\ Mpc$ from the symmetry centre. The orientation of the structure with respect to the line of sight is crucial to calculate $\mid \vec{v}_{r0} \mid$ at the observer position. In the absence of spherical symmetry, a certain velocity, e.g. $\mid \vec{v}_{r0} \mid = 500\ Km/s$, can be produced by objects having different masses and sizes (depending on the orientation), which would produce different effects on the CMB. Ellipsoidal homogeneous models were used previously by Atrio-Barandela & Kashlinsky (1992) and Chodorowski (1994) to study the anisotropies produced by pancake-like structures. The ellipsoids considered in this paper are inhomogeneous.

This paper is organized as follows. In Section 2, spherical, elongated, and flattened GAL structures are described. All of them are normalized according to Lynden-Bell et al. (1988). Anisotropies are calculated in Section 3. Calculations are based on the potential approximation; namely, on Eqs (1)–(3). First, results corresponding to the spherically symmetric case are compared to those obtained with the Tolman-Bondi solution for the same model. In this way, the approach based on Eqs. (1)–(3) is tested and the interpretation of the resulting anisotropy –as produced by a varying gravitational potential– is verified. The effects of the deformation with respect to spherical symmetry are then analyzed. Main conclusions and a general discussion are summarized in Section 4.

2 GREAT ATTRACTOR-LIKE MODELS

Whatever the spatial symmetry of the GAL structure may be, the following normalization condition is assumed: *At present time, there are points located at $43h^{-1}$ Mpc from the GAL centre where $\mid \vec{v}_{r0} \mid = 500$ Km/s.* We are interested in this low velocity a little smaller than the minimum velocity claimed by Linden-Bell et al (1988) because such a velocity allows us to obtain lower limits to the anisotropy produced by admissible GAL structures.

In what is called our spherical main (SM) model, initial conditions are chosen in the same way as in previous calculations based on the Tolman-Bondi solution. These conditions are set at redshift $z_i = 1000$. As in Arnau, Fullana & Sáez (1994), the initial profile of the density contrast is

$$\delta_I = \frac{\rho_I - \rho_{BI}}{\rho_{BI}} = \frac{\epsilon_1}{1 + (R/R_1)^2} + \frac{\epsilon_2}{1 + (R/R_2)^2} \ , \qquad (4)$$

where R is a radial coordinate and the parameters ϵ_1 (ϵ_2) and R_1 (R_2) set the amplitude and the size of a central overdensity (surrounding underdensity). The conditions $\epsilon_1 > 0$, $\epsilon_2 < 0$, $\epsilon_1 > \epsilon_2$, and $R_2 > R_1$ must be satisfied. The initial peculiar velocity is that corresponding to the above density profile in the case of vanishing nongrowing modes. In the SM model, the above parameters take on the values $\epsilon_1 = 5.42 \times 10^{-3}$, $\epsilon_2 = -6.7 \times 10^{-4}$, $R_1 = 4.21 \times 10^{-2} h^{-1} Mpc$, and $R_2 = 1.2 \times 10^{-1} h^{-1} Mpc$. In this case, the above normalization condition is satisfied and compensation is achieved at the last scattering surface of an observer placed at the GAL centre. The present density profile of the resulting structure was given in Arnau, Fullana, Sáez (1994, see curve

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C2 of Fig. 2). From this Figure, it follows that the contrast reduces to one-half of the central maximum value at $9h^{-1}$ Mpc. This profile shows a strong resemblance with the Great Attractor described by Linden-Bell et al. (1988). In spite of the fact that the exact compensation takes place very far from the symmetry centre, the present density contrast decreases fastly and, consequently, the energy density is negligible at distances much smaller than that of exact compensation. The SM model plays the role of an important reference, which has been well studied without approximating conditions; namely, with an exact cosmological solution of Einstein equations

In order to model GAL structures without spherical symmetry, the following initial density contrast has been chosen:

$$\delta_I = \frac{\epsilon_1}{1 + \sum_{i=1}^{3} X_i^2/A_{1i}^2} + \frac{\epsilon_2}{1 + \sum_{i=1}^{3} X_i^2/A_{2i}^2} \; , \tag{5}$$

where $X_i = ax^i$. The quantities A_{1i} and A_{2i} satisfy the following relations:

$$A_{2i} = mA_{1i} \; , m > 1 \; , \tag{6}$$

$$A_{11} = A_{12} = A_1 \; , A_{13} = nA_1 \; , \tag{7}$$

where A_1, m and n are free parameters defining the shape of the isodensity surfaces, which are ellipsoids. For $n = 1$, the profile (5) is spherically symmetric (S profiles). In the case $n > 1$, the isodensity surfaces are ellipsoids elongated (E profiles) along the x_3 direction. For $n < 1$, these surfaces are pancake-like (P profiles) ellipsoids. Directions x_1 and x_2 are undistinguisable. The structure is either elongated or flattened along the x_3 direction. These criteria about the names and characteristics of the axis must

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be taken into account carefully in order to imagine the orientations of the ellipsoids defined below.

The profile (5) has two appropriate features: (1) In the spherically symmetric limit $n = 1$, if one takes $A_{1i} = R_1$, $A_{2i} = R_2$ for $i = 1, 2, 3$, it reduces to the profile (4), which seems to have an admissible dependence on R, in agreement with Linden-bell et al. suggestions based on the peculiar velocity field (1988), and (2) in spite of the multidimensional features of the mass distribution, the gravitational potential corresponding to this profile can be calculated after performing a one-dimensional integral. From Binney and Tremaine (1987) and references cited therein, it can be proved easily that, inside the density distribution, the gravitational potential is of the form

$$\phi(x^i) = -\frac{1}{8} \frac{B_2 B_3}{B_1} \int_0^\infty \frac{\psi(\infty) - \psi(m)}{[(B_1^2 + \tau) + (B_2^2 + \tau) + (B_3^2 + \tau)]^{1/2}} d\tau \tag{8}$$

where $\psi(m)$ is defined as follows

$$\psi(m) = \int_0^{m^2} \delta_I(m^2) d(m^2) \tag{9}$$

and

$$m^2 = B_1^2 \sum_{i=1}^{3} \frac{(x^i)^2}{B_i^2 + \tau} \tag{10}$$

The parameter τ defines isopotential surfaces and the quantities B_i are proportional to A_i and define the boundary of the density distribution. We have taken a large enough value of the proportionality constant between B_i and A_i; thus, we are always concerned with the potential inside the distribution and Eq. (8) applies. In

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the asymmetric case, the gravitational potential is calculated by using the following

formula:

$$\phi(x^i, t) = \frac{3}{2}H^2 a^2 \Omega D \phi(x^i) \ , \tag{11}$$

where $\phi(x^i)$ is given by Eq. (8), and D is the growing mode of the density contrast

given by the following equations (Peebles, 1980):

$$D = 1 + \frac{3}{y} + \frac{3(1+y)^{1/2}}{y^{3/2}} ln[(1+y)^{1/2} - y^{1/2}] \ , \tag{12}$$

$$y = a\frac{H_0(1-\Omega_0)^{3/2}}{\Omega_0} \ . \tag{13}$$

The normalization of the GAL objects is achieved by using the formula

$$v^j = -\frac{2}{3H\Omega D}\frac{dD}{da}\frac{\partial\phi(x^i, t)}{\partial x^j} \tag{14}$$

The potential given by Eq. (11) can be obtained by using Eq. (3) and the linearized

density contrast $\delta = D\delta_I$. The application of the above linearized equations requires

discussion. Since normalization is based on the estimate of a present velocity and

the Great Attractor is not a linear structure at present (see Fullana, Sáez & Arnau

1994, for an estimate of the amplitude of the density contrast at t_0), the following

question arises: can we use Eqs. (11)–(14) for normalization?. In order to answer this

question we have developed a test. In the spherically symmetric case, the velocity

predicted by Eqs. (11)–(14) has been compared with that calculated with the Tolman

Bondi solution (exact nonlinear estimate). The resulting present velocity is 523 Km/s

to be compared with the velocity of 500 Km/s given by Tolman-Bondi calculations;

hence, the relative error given by the above approach –in the present peculiar velocity

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and, consequently, in the present spatial gradients of the gravitational potential– is $\sim 5\%$. This is in agreement with the known ansatz that velocities keep linear after the density contrast becomes nonlinear. Another important question is: can we use Eqs. (11)–(14) plus Eq. (1) in our computations of secondary gravitational anisotropies produced by GAL objects ($z = 5.9$, $\Omega_0 = 0.2$)?. According to Eq. (1), these anisotropies depend on the gradients of the gravitational potential along the photon null geodesics. The most relevant gradients are those calculated near the GAL object –at redsifht ~ 5.9– when the amplitude of the density contrast was close to unity. These gradients are calculated very near the linear regime and, consequently, their errors should be smaller than those of the present gradients estimated above (a few per cent). This means that only small errors should appear in our estimations based on the Potential Approximation plus Eq. (11). This is also confirmed by some nonlinear estimates presented below.

In order to normalize E and P profiles according to the criterium described above, the orientation of the GAL structure is crucial. This is because the peculiar velocity at the observer position depends on this orientation strongly. For each profile, two limit orientations are considered in which $\mid \vec{v}_{r0} \mid$ takes on its limit values. In the first (second) case, the line joining the observer and the GAL centre is parallel (orthogonal) to the x_3 axis. Thus, four normalizations are distinguised. Hereafter, these normalizations are denoted EP, EO, PP and PO. The first letter indicates the type of profile (Elongated or Pancake-like ellipsoids), while the second letter tell us in an

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evident manner whetter the line of sight of the GAL centre is either parallel (P) or orthogonal (O) to the x_3 axis.

Table 1 shows the values of the parameters defining the initial density contrasts of the Great Attractor realizations studied in detail in this paper. In all the cases, the values of the parameter m is 2.85. This value is that of the SM model used as a reference. For each pair (n, A_1), the parameters ϵ_1 and ϵ_2 have been obtained from the conditions of normalization and compensation. Compensation is achieved in such a way that it occurs at a present isodensity surface with a semiaxis of $9260h^{-1}$ Mpc. This is the distance from an observer located at the Great Attractor centre to his last scattering surface at $z = 1000$. In the spherically symmetric case, this compensation reduces to that used in the Tolman-Bondi treatment of the SM model.

3   RESULTS

All the selected GAL structures are located far from the last scattering surface and, consequently, they produce negligible temperature fluctuations and Doppler shifts on this surface; in which, the temperature is assumed to be constant. Furthermore, all the GAL objects are also placed far from the observer and, consequently, they produce negligible peculiar velocities at the observer position. This means that the Doppler kinematic dipole and quadrupole produced by the peculiar velocity of the observer are negligible. Finally, the Sunyaev-Zeldovich effect (see Sunyaev and Zel'dovich 1980 for a review) is not considered at all; hence, we are estimating a pure gravitational effect

produced far from the last scattering surface and, consequently, we are concerned with secondary gravitational anisotropies.

In the general case, for each normalization, the resulting anisotropy depends on both the location of the GAL centre and the orientation of its axis. This orientation corresponds to a structure located at redshift $z = 5.9$ and, consequently, it is absolutely independent on that considered for normalization, which corresponds to another object located at $43h^{-1}\ Mpc$ from the observer. For each normalization, two limit orientations – at $z = 5.9$ – are considered. These orientations correspond to the cases in which the line of sight of the GAL centre is Parallel (P) and Orthogonal (O) to the x_3 axis. The following notation is used: For a given normalization, for example EO, two estimations of the anisotropy are presented, which correspond to the orientations O and P defined above. Thus, we distinguish eight cases: EPP, EPO, EOO, EOP, POO, POP, PPP, PPO.

In the spherically symmetric case, the CMB anisotropy produced by a certain GAL structure has been computed by using three different codes. One of them is based on the Tolman-Bondi solution (see Arnau Fullana & Sáez 1994), the second one uses a linearized approach based on Eqs. (1)-(3) and (11) and, the third code uses Eqs. (1)-(3) and the density contrast $\delta = D\delta_I + D^2(\frac{5}{7}\delta_I^2 + \nabla\delta_I \cdot \nabla\Phi + \frac{2}{7}\Phi_{,ij}\,\Phi^{,ij})$ with $\nabla^2\Phi = \delta_I$ (see Sanz et al. 1996 for comments and references). This contrast includes a second order term. All the codes numerically compute the temperature T of the microwave background as a function of the observation angle ψ; this is the angle formed by

the line of sight and the line joining the observer and the inhomogeneity centre. In order to facilitate comparisons with Arnau, Fullana & Sáez (1994), the function $T(\psi)$ is then used to calculate the mean temperature $<T> = (1/2) \int_0^\pi T(\psi) \, sin\psi \, d\psi$ and the total temperature contrast $\delta_{GRAV}(\psi) = [T(\psi) - <T>] / <T>$ and, finally, second-order radial differences at the angular scale $\alpha = 8.1°$ are calculated, where α stands for the angle between two observational directions. These second order differences are defined as follows

$$(\Delta T/T)_{8.1}(\psi) = \frac{1}{2}\{[\delta_{GRAV}(\psi) - \delta_{GRAV}(\psi - 8.1)] - [\delta_{GRAV}(\psi + 8.1) - \delta_{GRAV}(\psi)]\} \quad (15)$$

Figure 1 shows the second order differences $(\Delta T/T)_{8.1}(\psi)$ corresponding to the SM model of reference. The dashed line displays the results given by calculations based on the Tolman-Bondi solution, while the solid line has been obtained from Eqs. (1)-(3) and (11). For these two lines, the relative error in the amplitudes appears to be ~ 30 per cent. The angular scales are similar. The errors appearing as a result of the use of Eq. (11) –linear estimation of the gravitational potential– are expected to be smaller than a few per cent (see Section 2). This expectation about the smallness of the errors produced by linearization is confirmed by the comparison of the solid and dotted lines, which have been obtained from density contrasts approximated up to first and second order, respectively. These two lines are almost undistinguishable in the Figure. An appropriated zoom has been included to display the differences. The relative difference between the amplitudes is 2.3 % and, consequently, the effect of the nonlinear term of the density contrast cannot be the main source of the ~ 30 per cent

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error mentioned above. The main part of this error should be due to the presence of large spatial scales leading to significant curvature effects. Are these spatial scales associated to the density contrast as a result of our compensations at large distances from the symmetry centre?. In order to test this possibility, compensations at much smaller distances from the symmetry center have been performed and errors near 30% have been obtained in all the cases. This means that the large scales involved in the problem correspond to the large regions where the gravitational potential is significant.

The above discussion about the nonlinear effect justifies the use of the linear approach in the following applications.

The size of spherical GAL structures has been varied maintaining the model of Section 2 and its normalization. In order to vary the size, the value of A_1 corresponding to the SM model of Figure 1 (first row of Table 1) has been multiplied by the factor ξ, while parameters m and n have not been altered. The parameter ξ, which set the GAL size, has been varied in the interval [0.5,2]. The anisotropy produced by each of the resulting structures has been estimated. Figure 2 displays the relation between the amplitude of the $(\Delta T/T)_{8.1}(\psi)$ differences and the size of the GAL structure defined by factor ξ. The relation $(\Delta T/T)_{8.1}(0) = 4.947 \times 10^{-6} - 2.439 \times 10^{-6}\xi + 1.979 \times 10^{-5}\xi^2$ fits very well the numerical estimates (stars). It is noticeable that the CMB anisotropy decreases as the GAL size decreases. This means that a too concentrated GAL structure would not produce any significant effect. Linden-Bell, et al (1988) claimed that

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the Great Attractor core has a radius $R_c \sim 10h^{-1}\ Mpc$ and a $1/R^2$ density profile. In this case, it would produce anisotropies at the level 10^{-5}; nevertheless, more observational data have been and are being obtained (Kraan-Korteweg, Woudt & Henning, 1996). Future conclusions about the size and mass of the Great Attractor would be important in order to get a definitive estimation of its contribution to the CMB anisotropy in open universes.

In the spherically symmetric case, all the planes containing the line of sight of the symmetry centre are equivalent. In the absence of spherical symmetry, we present $(\Delta T/T)_{8.1}(\psi)$ differences in two planes. Each of these planes is generated by the line of sight of the symmetry centre and one of the ellipsoid axis perpendicular to this line. They are two orthogonal planes. For a fixed normalization, if the axis perpendicular to the line of sight are the undistinguishible axis x_1 and x_2, the second order differences corresponding to both planes should coincide. On the contrary, for the pairs (x_1, x_3) and (x_2, x_3), the resulting differences are expected to be different. The differences between the results corresponding to both planes are due to the asymmetry of the GAL object which has been located and orientated at $z = 5.9$.

Figure 3 shows the second order differences $(\Delta T/T)_{8.1}(\psi)$ for the eight cases defined in Section 2. Table 2 gives the values of the maxima, $(\Delta T/T)_{8.1}(0)$, appearing in the curves of this Figure. The chosen cases correspond to various profiles, normalizations and orientations of the symmetry axis at $z = 5.9$. In each panel, the type of profile (first letter inside the panel: E or P), the normalization (second letter: P

or O) and the orientation at $z = 5.9$ (third letter P or O) have been fixed. Continuous and dashed lines display second order differences in each of the two orthogonal planes defined above. As expected, when the axis perpendicular to the line of sight are x_1 and x_2 (the third letter is a P), both curves coincide. In the remaining cases, the continuous and dashed lines are different but rather similar. The two top panels (fourth level in the Figure) correspond to E profiles normalized in the same way, but having a different orientation at $z = 5.9$. The amplitude of the left (right) panel is $\sim 5.7 \times 10^{-5}$ ($\sim 4.8 \times 10^{-5}$); hence, the orientation at $z = 5.9$ is not very important. It leads to deviations of about 15 per cent. The same conclusion is obtained from comparisons of the left and right panels in the third, the second and the first level of panels (see Table 2 for the values of the amplitudes). Comparisons of the panels corresponding to distinct levels show the importance of the profile and the normalization. For E (P) profiles, the differences between the third (first) and the fourth (second) levels of panels appear as a result of normalization. We can conclude than the normalization –more properly the orientation defining the normalization– is very important. It can modify the resulting anisotropy by a factor ~ 2. As it can be seen in Table 2, the smallest (greatest) amplitude appears in the case EOO (POO) and its value is $\sim 2.5 \times 10^{-5}$ ($\sim 7.0 \times 10^{-5}$). In any case, the resulting anisotropy is very significant.

In Figure 4, we have considered a EPP case with the semi-axis reduced by a factor 1/2 with respect to those of the EPP realization of Figure 3. As in the spherically

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symmetric case, we can see that the CMB anisotropy has decreased. The amplitude corresponding to Fig. 4 is 1.34×10^{-5}.

4   CONCLUSIONS AND DISCUSSION

In this paper, the following elements have been fixed: a normalization condition for GAL structures (first paragraph of Section 2), a form for the initial density profile, a compensation distance, an open background ($\Omega_0 = 0.2$), and a redshift for location ($z = 5.9$). The resulting GAL structures are very similar to that described by Linden-Bell et al. (1988).

The spherically symmetric case has been analyzed in detail. The use of both the potential approximation and the Tolman-Bondi solution has proved that, for $z = 5.9$ and $\Omega_0 = 0.2$, objects having cores with present sizes near a ten of Megaparsecs produce significant anisotropies with amplitudes of 10^{-5} on scales of various degrees. This is because the gradients of the gravitational potential are significant on spatial scales of a few hundred of Megaparsecs (at z=5.9) subtending angular scales of various degrees; however, the angle subtended by the central region where the density contrast is significant is smaller than the angular scale of the anisotropy.

Starting from the same initial conditions (first row of Table 1), computations based on both the Tolman-Bondi solution and Eqs (1)–(3) plus (11) have given comparable anisotropies with very similar scales and a little different amplitudes. It is noticeable that the relative differences between the resulting amplitudes – ~ 0.3 – is very

similar to the ratio between the scales where the CMB photons feel the variations of the gravitational potential and the curvature scale. Since the errors due to the use of Eq. (11) should justify only a small part of these relative differences, the most important part of them should appear as a result of the application of the potential approximation to extended regions where the curvature should be taken into account rigorously.

Finally, in the spherically symmetric case with the profile (4), it has been proved that the anisotropy produced by GAL objects depends strongly on their size. This dependence was not pointed out in previous papers based on the Tolman-Bondi solution. In spite of the fact that the normalization has not been changed, the amplitude of the resulting anisotropy and its angular scale decreases as the size of the object decreases.

In the case of the pancake-like and ellipsoidal-like GAL structures with the profile (5), the angular scales have appeared to be very similar to those of the spherically symmetric case, but the amplitudes are rather different. In some cases, the amplitude of ellipsoidal structures become magnified by a factor near 3 with respect to the case of spherical objects. Furthermore, it has been verified that: (a) the amplitudes depend on the orientation of the axis at $z = 5.9$, but this dependence is weak, (b) the orientation assumed for normalization is very important because it conditionates the size and mass of the resulting structure and, consequently, the anisotropy amplitude, and (c) the amplitude and the angular scale of the anisotropy depend on the spatial

size of the structure as it occurs in the spherically symmetric case.

The anisotropy produced by all the GAL objects of the Universe essentially depends on: the features of the Great Attractor (normalization and size), the value of the density parameter and the abundance of this kind of objects. New observations giving information about some of these elements would be necessary in order to obtain definitive conclusions from Fig. 3. In fact, if the value of the density parameter is found to be $\Omega_0 = 0.2$ and various GAL objects are observed between $z = 2$ and $z = 30$ (see Arnau, Fullana, Sáez 1994), data from C.O.B.E. satellite and TENERIFE experiment would rule out various GAL realizations of Fig. 3. In fact, these experiments give amplitudes $\sim 10^{-5}$ for angular scales of a few degrees and, consequently, realizations EPP, EPO, POO and POP would be inadmissible. According to the same experiments, if the Great Attrator is found to be an EPP realization and GAL objects are abundant enough, the density parameter $\Omega_0 = 0.2$ is forbidden. On the contrary, if the universe is found to be quasi-flat, no conclusions would be obtained from Fig. 3 because any GAL objects would produce negligible anisotropies.

The fact that the angular scales appear to be of various degrees –in all the cases considered in this paper– should be taken into account in other types of usual calculations. Let us discuss this point in more detail. Suppose that we take a Fourier box where a certain realization of structures is generated. If we wish to estimate –numerically– the anisotropies appearing in a universe filled by these boxes, only the effect of structures much smaller than the size of the box can be taken into ac-

count. This is a well known fact, which must be analyzed in the case of the secondary anisotropies given by Eq. (1); in this case, according to our conclusions, the box should be much greater than the regions where the variations of the potential are significant (not greater than the density structures). This means that calculations of the effect produced by various possible realizations of the Great Attractor would require too big boxes with a huge size of thousands of Megaparsecs. In calculations based on Eqs. (1)–(3) plus spectra and statistics, it should be taken into account that: (1) these equations lead to substantial error in the presence of big structures as the Great attractor (~ 30 per cent), and (2) the usual spectra and statistics and specially their time evolution could require substantial modifications in order to account for the presence and evolution of GAL objects.

References

Arnau J.V., Fullana M.J., & Sáez D., 1994, MNRAS, 268, L17

Atrio-Barandela F. & Kashlinsky, A., ApJ, 1992, 390, 322

Binney, J., & Tremaine, S., 1987, *Galactic Dynamics.* Princeton Series in Astro-

physics. Princeton University Press

Bondi H., 1947, MNRAS, 107, 410

Chodorowski, M., 1994, MNRAS, 266, 897

Fullana M.J., Sáez D. & Arnau J.V., 1994, ApJ Supplement, 94, 1

Kraan-Korteweg R.C., Woudt P.A., & Henning P.A., 1996, astro-ph/9611099

Linden-Bell D., Faber S.M., Burstein D., Davies R.L., Dressler A., Terlevich R.J., Wegner G., 1988, ApJ, 326, 19

Martínez-González E., Sanz J.L. & Silk J., 1990, ApJ, 355, L5

Panek M., 1992, ApJ, 388, 225

Peebles P.J.E., 1980, *The Large Scale Structure of the Universe.* Princenton University Press

Rees M.J. & Sciama D.W., 1968, Nat, 217, 511

Sanz J.L., Martínez-González E., Cayón L., Silk J. & Sugiyama N., 1996, ApJ, 467, 485

Sachs R.K., & Wolfe A.M., 1967, ApJ, 147, 73

Sáez D., Arnau J.V. & Fullana M.J., 1993, MNRAS, 263, 681

Sáez D., Arnau J.V. & Fullana M.J., 1995, Astro. Lett. Communications, 32, 75

Sunyaev R.A. & Zel'dovich Ya. B., 1980, A&A. 18, 537

Tolman R.C., 1934, Proc. Natl Acad. Sci., 20, 169

Tuluie R., Laguna P. & Anninos P., 1996, ApJ, 463, 15

Figure Captions

Fig. 1. Second order differences $(\Delta T/T)_{8.1}$ as functions of the observation angle ψ in degrees for the spherically symmetric GAL structure corresponding to the first row of Table 1 (SM model). Continuous, dotted and dashed lines correspond to estimates based on a linear approach, a second order approach and the Tolman-Bondi solution, respectively. The zoom magnifies the region around $\psi = 0$. Its axis show the same quantities as those of the main Figure.

Fig. 2. Amplitude of $(\Delta T/T)_{8.1}$ as a function of the parameter ξ defining the size of the GAL structure. Stars show the amplitudes numerically estimated and the continuous line is a good fit to these values.

Fig. 3. Same as Fig. 1. Each panel correponds to one of the cases EPP, EPO, EOO, EOP, POO, POP, PPP, PPO defined in the text. Continuous and dashed lines show $(\Delta T/T)_{8.1}$ differences in the planes (x_1, x_2) and (x_1, x_3), respectively. The axis x_1 has the direction of the line of sight. In - - P (- - O) cases, both curves are identical (different).

Fig. 4. Same as in the top left panel of Fig. 3, but the semiaxis of the ellipsoids have been reduced by the factor $1/2$.

Table 1

Realizations of the Great Attractor

Profile	Orientation	ϵ_1	ϵ_2	A_1 $(h^{-1}\ Mpc)$	n
S	–	5.42×10^{-3}	-6.7×10^{-4}	4.21×10^{-2}	1
S	–	1.19×10^{-2}	-1.49×10^{-3}	2.1×10^{-2}	1
E	P	7.15×10^{-3}	-9.04×10^{-4}	4.21×10^{-2}	2
E	O	4.06×10^{-3}	-5.28×10^{-4}	4.21×10^{-2}	2
P	O	5.17×10^{-3}	-6.54×10^{-4}	8.42×10^{-2}	0.5
P	P	3.06×10^{-3}	-3.97×10^{-4}	8.42×10^{-2}	0.5
E	P	1.07×10^{-2}	-1.34×10^{-3}	2.1×10^{-2}	2

Table 2

Anisotropies of ellipsoidal Great Attractor-like objects

CASE	PLANE ..	$(\Delta T/T)_{8.1}(0)$	CASE	PLANE	$(\Delta T/T)_{8.1}(0)$
EPP	(x_1, x_3)	5.68×10^{-5}	EPP	(x_2, x_3)	5.68×10^{-5}
EPO	(x_1, x_2)	4.86×10^{-5}	EPO	(x_1, x_3)	4.80×10^{-5}
EOO	(x_1, x_2)	2.46×10^{-5}	EOO	(x_1, x_3)	2.52×10^{-5}
EOP	(x_1, x_3)	2.83×10^{-5}	EOP	(x_2, x_3)	2.83×10^{-5}
POO	(x_1, x_2)	6.93×10^{-5}	POO	(x_1, x_3)	7.04×10^{-5}
POP	(x_1, x_3)	6.08×10^{-5}	POP	(x_2, x_3)	6.08×10^{-5}
PPP	(x_1, x_3)	3.37×10^{-5}	PPP	(x_2, x_3)	3.37×10^{-5}
PPO	(x_1, x_2)	3.81×10^{-5}	PPO	(x_1, x_3)	3.76×10^{-5}

$\Delta T/T_{(8.1)}(0)$

8×10^{-5}

6×10^{-5}

4×10^{-5}

2×10^{-5}

0

0.5　　1.0　　1.5　　2.0

ξ